Solutions for the Homework 2

Problem 1.11: A galaxy in the constellation Ursa Major is receding from the earth as $15,000km/s$. If one of the characteristic wavelengths of the light the galaxy emits is $550nm$, what is the corresponding wavelength measured by astronomers on the earth?

Solution: Use the our Doppler effect formula with positive sign for receding direction. (Eq. 1.6)

$$\lambda \ = \ \frac{c}{\nu} \ = \ \frac{c}{\nu_0}\sqrt{\frac{1+v/c}{1-v/c}} \ = \ \lambda_0\sqrt{\frac{1+v/c}{1-v/c}} \ = \ 578nm.$$

Detail calculation for λ (link)

Problem 1.16: (a) Show that when $v \ll c$, the formulas for the doppler effect both in light and in sound for an observer approaching a source, and vice versa, all reduce to $\nu \approx \nu_0(1+v/c)$, so that $\delta\nu/\nu \approx v/c$. [Hint: For $x \ll 1$, $1/(1+x) \approx 1-x$.] (b) What do the formulas for an observer receding from a source, and vice versa, reduce to when $v \ll c$?

Solution: (a) For non-relativistic case, suppose that V is a velocity of source and positive sign for the direction to observer. And also define the sign of v is positive when observer approaching to sources. (See Eq. 1.4) Then, there are two cases. $v = 0$ and $V \ll C$ for moving source case and $V = 0$ and $v \ll c$ for moving observer case but each cases reduces to the same form up to first order of v/c.

$$\begin{aligned}\nu \ &= \ \nu_0\left(\frac{1+v/c}{1-V/c}\right) \ = \ \nu_0\left(\frac{1}{1-V/c}\right) \ \approx \ \nu_0(1+V/c),\\ \nu \ &= \ \nu_0\left(\frac{1+v/c}{1-V/c}\right) \ = \ \nu_0(1+v/c).\end{aligned}$$

For relativistic case, we don't need to split the cases of moving source and observer, so use the Eq. 1.7 with the condition $v \ll c$. Then,

$$\nu \ = \ \nu_0\sqrt{\frac{1+v/c}{1-v/c}} \ \approx \ \nu_0(1+v/2c)^2 \ \approx \ \nu_0(1+v/c),$$

where this result is same for non-relativistic case.

(b) When the observer receding from the light and sound source, the case is same as the previous case with only different sign. So the result will be

$$\nu \ = \ \nu_0(1-v/c).$$

Problem 2.9: Light from the sun arrives at the earth, an average of $1.5 \times 10^{11} m$ away, at the rate of $1.4 \times 10^3 W/m^2$ of area perpendicular to the direction of the light. Assume that sunlight is monochromatic with a frequency of $5.0 \times 10^{14} Hz$. (a) How many photons fall per second on each square meter of the earth's surface directly facing the sun? (b) What is the power output of the sun, and how many photons per second does it emit? (c) How many photons per cubic meter are there near the earth?

Solution: (a) Let the number of falling photons per second per square meter as l. Then,

$$l = \frac{\text{Power per area}}{h \times \text{Frequency of each photon}} = 4.2 \times 10^{21}/s \cdot m^2.$$

Detail calculation for l (link)

(b) Since we know the power per area at earth orbit, we can get the power output of the sun as follows:

$$P = \text{Power per area} \times \text{Area of earth orbital sphere} = 4.0 \times 10^{26} W,$$

and the number of photons per second emitted, m as follows:

$$m = P/(h \times \text{Frequency of each photon}) = 1.2 \times 10^{45}/s.$$

Detail calculation for P (link)
Detail calculation for m (link)

(c) Let the number of photons as n. From (a), there are 4.2×10^{21} photons in the volume $c \times 1s \times 1m^2$. So,

$$n = \text{Result of (a)}/c = 1.4 \times 10^{13}/m^3.$$

Detail calculation for n (link)

Problem 2.21: Electrons are accelerated in television tubes through potential differences of about $10kV$. Find the highest frequency of the electromagnetic waves emitted when these electrons strike the screen of the tube. What kind of waves are these?

Solution: The energy of electron is $e \times 10kV$ and it is same as the possible maximum energy of electromagnetic waves. So, the highest frequency of electromagnetic wave is

$$\nu = \frac{eV}{h} = 2.4 \times 10^{18} Hz,$$

which corresponds to x-ray. (See Fig. 2.2)

Detail calculation for ν (link)

Problem 2.27: In Sec. 2.7 the x-rays scattered by a crystal were assumed to undergo no change in wavelength. Show that this assumption is reasonable by calculating the Compton wavelength of a Na atom and comparing it with the typical x-ray wavelength of $0.1nm$.

Solution: Let the mass of Na atom as M. To get the Compton wavelength, follow the Sec. 2.7 and replace the mass of electron to Na atoms. Then,

$$\lambda_C = \frac{h}{Mc} = 5.7896 \times 10^{-17} m.$$

This is much less than the given typical x-ray wavelength.

Detail calculation for λ_C (link)

Problem 2.35: A photon of frequency ν is scattered by an electron initially at rest. Verify that the maximum kinetic energy of the recoil electron is $KE_{\max} = (2h^2\nu^2/mc^2)/(1 + 2h\nu/mc^2)$.

Solution: Follow the Sec. 2.7 and from the Eq. 2.20 with $\phi = 180°$,

$$KE_{\max} = \frac{2(h\nu)(h\nu')}{mc^2}.$$

Since $\nu'/\nu = \lambda/\lambda'$,

$$\nu' = \nu\left(\frac{\lambda}{\lambda'}\right) = \nu\left(\frac{\lambda}{\lambda + \Delta\lambda}\right) = \nu\left(\frac{\lambda}{\lambda + 2h/mc}\right) = \frac{\nu}{1 + 2h\nu/mc^2},$$

and using this relation,

$$KE_{\max} = \frac{2h^2\nu^2/mc^2}{1 + 2h\nu/mc^2}.$$

Problem 2.39: A positron collides head on with an electron and both are annihilated. Each particle had a kinetic energy of $1.00MeV$. Find the wavelength of the resulting photons.

Solution: This process make two photons. Since total energy of electron and positron is $2 \times 1.00MeV + 2 \times 0.511MeV = 3.02MeV$. (Rest energy of electron is $0.511MeV$.) Then, the wavelength of photon is

$$\lambda = \frac{2hc}{3.02MeV} = 8.21 \times 10^{-13} m.$$

Detail calculation for λ (link)

Problem 2.40: A positron with a kinetic energy of $2.000MeV$ collides with an electron at rest and the two particles are annihilated. Two photons are produced; one moves in the same direction as the incoming positron and the other moves in the opposite direction. Find the energies of the photons.

Solution: Suppose that the electron mass is m, the total energy of positron is $E = 2.511MeV$ and each photon have an energy E and a momentum p with indices 1 for same direction and 2 for opposite direction as the incoming positron. Then,

$$p = p_1 - p_2 \Rightarrow pc = E_1 - E_2,$$

from the momentum conservation and

$$E + mc^2 = E_1 + E_2,$$

from the energy conservation. Using these two equations,

$$E_1 = \frac{E + mc^2 + pc}{2} = \frac{E + mc^2 + \sqrt{E^2 - m^2c^4}}{2} = 2.740MeV,$$
$$E_2 = E + mc^2 - E_1 = 0.282MeV.$$

Detail calculation for E_1 (link)

Problem 2.42: (a) Verify that the minimum energy a photon must have to create an electron-positron pair in the presence of a stationary nucleus of mass M is $2mc^2(1+m/M)$, where m is the electron rest mass. (b) Find the minimum energy needed for pair production in the presence of a proton.

Solution: (a) In the zero momentum frame, there are electron, positron and nucleus. So, the total energy of this system is summation of all rest energy and kinetic energy, KE. Then the invariant mass $\mathcal{M}$ is $\mathcal{M} = ((2m + M)c^2) + KE)^2$. Now, consider the given situation in the problem. In this frame, invariant mass will be same as the zero momentum case. From this relation,

$$(pc + Mc^2)^2 - p^2c^2 \;\geq\; (2m + M)^2c^4 \quad \Rightarrow \quad pc \;\geq\; 2mc^2(1 + m/M).$$

So, the minimum energy a photon must have is $2mc^2(1 + m/M)$.

(b) Generally, $M \gg m$ and so the minimum energy needed is

$$pc \;\approx\; 2mc^2 \;=\; 1.022MeV.$$